Theoretical Perspectives on Flavor Physics and CP violation

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• Introduction to flavor physics
  ... Why bother? What are we after?

• Status of CKM matrix
  ... Why are $\sin 2\beta$ and $B_{d,s}$ mixing clean?

• Future: what are sensitive tests?
  ... Some nice and clean measurements
  ... Bits of theory: SCET

• Conclusions
Why is flavor physics and CPV interesting?

- Almost all extensions of the SM contain new sources of $CP$ and flavor violation (e.g., 43 new CPV phases in SUSY [must see superpartners to discover it])

- A major constraint for model building
  (flavor structure: universality, heavy squarks, squark-quark alignment, ...)

- May help to distinguish between different models
  (mechanism of SUSY breaking: gauge-, gravity-, anomaly-mediation, ...)

- The observed baryon asymmetry of the Universe requires CPV beyond the SM
  (not necessarily in flavor changing processes in the quark sector)

There is no “standard” new physics scenario in flavor sector...
# Baryogenesis

\[ \frac{\text{# baryons}}{\text{# photons}} \sim 10^{-9} \quad \text{now} \iff \frac{n_q - n\bar{q}}{n_q + n\bar{q}} \sim 10^{-9} \quad \text{at } t < 10^{-6} \text{ sec } (T > 1 \text{ GeV}) \]

- To produce such an asymmetry, need (Sakharov conditions)
  1. baryon number violating interactions
  2. \(C\) and \(CP\) violation
  3. deviation from thermal equilibrium

- SM contains 1–3, but
  A. \(CP\) violation is too small
  B. deviation from thermal equilibrium too small with just one Higgs doublet

NP models can solve A–B near the weak scale, and may have observable effects (possibly only in flavor diagonal processes, such as electric dipole moments)
Two large mixing angles observed — a real surprise!

Leptogenesis appears more and more plausible:
... generate $B - L$ by CPV decay of $\nu_{\text{heavy}}$
... $\nu_{\text{heavy}}$ lives long enough to decay when $T < m_{\nu_{\text{heavy}}}$

Baryon asymmetry due to $B + L$ violating but $B - L$ conserving processes above electroweak phase transition

Model dependent whether relevant CPV parameters are related to CPV in light neutrino sector

Connection to TeV scale is model dependent
Central questions of flavor physics

1. Does the SM (only virtual quarks, $W$, and $Z$ interacting through CKM matrix in tree and loop diagrams) explain all flavor changing interactions?

2. At what level and where could we see deviations?

Need: experimental precision ($B$ factories) and theoretical precision (cleanliness)

New physics most likely to modify:
- SM loop processes: mixing
- SM loop processes: rare decays
- $CP$ violation

So we want to study:
- mixing & rare decays
- CPV asymmetries
- compare tree and loop processes

The point is not only to measure CKM elements, but to overconstrain the SM by many “redundant” measurements; correlations may be crucial to narrow down NP
The problem: strong interactions

- Can we learn about high energy physics from low energy hadronic processes?

Solutions:
- Symmetries of QCD (exact or approximate)
- Certain processes are determined by short-distance physics

Sometimes possible to combine data and symmetries to eliminate hadronic mess

Example: \( \sin(2\beta) \) from \( B \rightarrow \psi K_S \) — amplitude not calculable

Solution: \( CP \) symmetry of QCD (\( \theta_{QCD} \) can be neglected)

\[
\langle \psi K_S | \mathcal{H} | B^0 \rangle = - \langle \psi K_S | \mathcal{H} | \bar{B}^0 \rangle \times [1 + O(\alpha_s \lambda^2)]
\]

- The key processes are those which can teach us about high energy physics without hadronic uncertainties
Status of CKM matrix
Charged current weak interactions — CKM matrix:

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
\sim \begin{cases}
1 & \text{for } \lambda \\
\lambda & \text{for } \lambda^2 \\
\lambda^3 & \text{for } \lambda^3
\end{cases}
\]

Depend on 3 angles + 1 phase — only source of CPV in the SM (except for $\theta_{QCD}$)
CKM matrix and the unitarity triangle

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\]

\(\lambda \sim 0.22\)

Depend on 3 angles + 1 phase — only source of CPV in the SM (except for \(\theta_{QCD}\))

- The unitarity triangle provides a simple way to visualize the SM constraints

\[V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0\]

The angles and sides are directly measurable — want to overconstrain this picture
CKM matrix and the unitarity triangle

- Convenient to exhibit hierarchical structure by expanding in $\lambda = \sin \theta_C$

$$V = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Present uncertainties: $\lambda \sim 1\%, A \sim 5\%, \eta/\rho \sim 7\%, \sqrt{\rho^2 + \eta^2} \sim 20\%$

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- Constraints on CKM usually plotted on the $(\bar{\rho}, \bar{\eta})$ plane

Main uncertainties of two sides:

$V_{td}$: $B_s$ mixing or lattice $f_{B_d}^2 B_{B_d}$

$V_{ub}$: semileptonic $b \rightarrow u$ decays
Why $B$ physics?

- CPV in $K$ system is at the right level ($\epsilon_K$ can be described with $O(1)$ CKM phase); hadronic uncertainties preclude precision tests ($\epsilon'_K$ notoriously hard to calculate)

Plan to measure $K \to \pi\nu\bar{\nu}$ — theoretically clean, but $B \sim 10^{-10}(K^\pm)$, $10^{-11}(K_L)$

$$A \propto \begin{cases} 
(\lambda^5 m_t^2) + i(\lambda^5 m_t^2) & t: \text{CKM suppressed} \\
(\lambda m_c^2) + i(\lambda^5 m_c^2) & c: \text{GIM suppressed} \\
(\lambda \Lambda^2_{\text{QCD}}) & u: \text{GIM suppressed}
\end{cases}$$

- In $D$ decays the SM predicts small CPV — both GIM and CKM suppressed

- In the $B$ meson system, large variety of interesting processes:
  - top quark loops neither GIM nor CKM suppressed (large mixing, rare decays)
  - large $CP$ violating effects possible, some of which have clean interpretation
  - some of the hadronic physics understood model independently ($m_b \gg \Lambda_{\text{QCD}}$)
$B_{d,s}$ mixing and $\sin 2\beta$
**$B_{d,s}$ mixing:** $|V_{td}|$ and $|V_{ts}|$

Two mass eigenstates: $|B_{H,L}\rangle = p|b\bar{d}\rangle \mp q|\bar{b}d\rangle$

Mixing dominated by top quarks:

\[ (\bar{b}_L \gamma_\nu d_L)(\bar{b}_L \gamma^\nu d_L) \]

\[ \Delta m_q = 2|M_{12}| = |V_{tb}V_{tq}^*|^2 f_{B_s}^2 B_{B_q} \times [\text{known factors}] \]

Nonperturbative matrix element

In $SU(3)$ symmetry limit: $\xi^2 \equiv f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d} = 1$

Lattice QCD: $\xi^2 \sim [1.15(6)]^2$ \hspace{1cm} Chiral logs: $\sim 1.3$

Need more reliable control of light quark effects

This may soon be the main limitation to extract $|V_{td}/V_{ts}|$

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ZL — p. 8
CPV in interference between decay and mixing

- Especially interesting if both $B^0$ and $\bar{B}^0$ can decay to same final state, e.g., $|f\rangle = |f_{CP}\rangle$:

$$\lambda_{f_{CP}} = \frac{q}{p} \frac{A_{f_{CP}}}{A_{f_{CP}}^*} = \eta_{f_{CP}} \frac{q}{p} \frac{A_{f_{CP}}}{A_{f_{CP}}^*}$$

$$a_{f_{CP}} = \frac{\Gamma[\bar{B}^0(t) \rightarrow f] - \Gamma[B^0(t) \rightarrow f]}{\Gamma[\bar{B}^0(t) \rightarrow f] + \Gamma[B^0(t) \rightarrow f]} = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2} \sin(\Delta m t) - \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta m t)$$

$CP$ violation: $|\lambda_f| \neq 1 \Rightarrow$ CPV in mixing and/or decay

$\text{Im} \lambda_f \neq 0 \Rightarrow$ CPV in interference

- If amplitudes with one weak phase dominate a decay then the CP asymmetry measures a phase in the Lagrangian theoretically cleanly

(Then $|\lambda_f| \simeq 1$, since $|q/p| - 1 < \mathcal{O}(10^{-2})$ in $B_{d,s}$ mixing)

$$a_{f_{CP}} = \text{Im} \lambda_f \sin(\Delta m t)$$
The cleanest case: $B \rightarrow \psi K_{S,L}$

- Several contributions:
  - “Tree” ($b \rightarrow c\bar{c}s$): $\overline{A}_T = \lambda^2 V_{cb} V_{cs}^* A_{c\bar{c}s}$
  - “Penguin”: $\overline{A}_P = \lambda^2 V_{tb} V_{ts}^* P_t + \lambda^2 V_{cb} V_{cs}^* P_c + \lambda^4 V_{ub} V_{us}^* P_u$

Write sum as:

$\overline{A}_{\psi K_S} = \lambda^2 V_{cb} V_{cs}^* [A_{c\bar{c}s} + P_c - P_t] + \lambda^4 V_{ub} V_{us}^* [P_u - P_t]$

- “Tree” phase suppressed by $\lambda^2$

- The $V_{cb} V_{cs}^*$ term dominates $\Rightarrow$ theoretically very clean

$\lambda_{\psi K_{S,L}} = \mp \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cb} V_{cs}} \right) \left( \frac{V_{cs} V_{cd}^*}{V_{cs} V_{cd}} \right) = \mp e^{-2i\beta} \Rightarrow \text{Im} \lambda_{\psi K_{S,L}} = \pm \sin 2\beta
Present knowledge of \((\bar{\rho}, \bar{\eta})\)

Standard model fit without \(\sin 2\beta\)

\[\begin{align*}
\rho & \quad \eta \\
\rho_{\text{fit}} & \quad \eta_{\text{fit}} \\
|V_{ub}/V_{cb}| & \quad \varepsilon_{K}
\end{align*}\]
Present knowledge of \((\bar{\rho}, \bar{\eta})\)

Full SM fit including \(\sin 2\beta\)

The CKM picture passed its first real test

Paradigm change: look for corrections, rather than alternatives \((\Delta m_{B_S}, S_{\phi K_S})?\)

Is the SM the only source of CPV?

Does the SM fully explain flavor physics?

Key measurements: theoretically clean and experimentally doable

Need others, besides \(\beta\) and \(|V_{td}/V_{ts}|\):

1) Model independent extraction of \(|V_{ub}|\)
2) Factorization — may help with \(\alpha, \gamma\)
3) “Zero prediction” observables
Future: what are the good tests?
What are we after?

- In SM: Only $V_{ub}$ and $V_{td}$ have large phases (in usual parameterization) any large interference type CPV is a function of these

  $\beta$ is “easy” to measure, second can be called: $\alpha$, $\gamma$, $\beta + \gamma$, $2\beta + \gamma$ ...

  but this does not make any difference

  Independent measurements are cross-checks

- Beyond SM: Many phases can be large and different ($B_{d,s}$ mixing, decays)

  “$\alpha$, $\beta$, $\gamma$” is only a language: measurements that relate to same angle in SM can be sensitive to different NP

  Independent measurements (which have clean interpretation) search for NP!
How to find new physics?

Q: Big deal... Do all possible tests
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A: Some tests are better than others
How to find new physics?

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A: Some tests are better than others

Q: It’s trivial... Check $\alpha + \beta + \gamma = \pi$
How not to find new physics?

Q: Big deal... Do all possible tests

A: Some tests are better than others

Q: It’s trivial... Check $\alpha + \beta + \gamma = \pi$

A: This is the wrong test

i) In most NP models $\alpha + \beta + \gamma = \pi$

ii) Even if $\alpha + \beta + \gamma \neq \pi$, probably an easier test will show NP first

iii) Takes very long time and hard to do
How can new physics show up?

1. Two measurements which relate to the same quantity in the SM incompatible, e.g., \( S_{\psi K_S} \neq S_{\phi K_S} \)

2. \( B_s \) or \( D \) mixing incompatible with SM, e.g., \( \Delta m_{B_s} \gtrsim 30 \text{ ps}^{-1} \)

3. Angles inconsistent with sides

4. Zero prediction observable found large, e.g., \( a_{CP}(B_s \to \psi\phi), a_{CP}(B \to s\gamma) \)

5. Enhancement of rare decays of \( B, B_s, K, D \)

All are easier than checking \( \alpha + \beta + \gamma = \pi \) and more sensitive to NP
Some nice and clean measurements
Amplitudes with one weak phase expected to dominate:

\[
\bar{A} = V_{cb}V_{cs}^* [P_c - P_t + T_{c\bar{c}s}] + V_{ub}V_{us}^* [P_u - P_t + T_{u\bar{u}s}]
\]

dominant contribution suppressed by \(\lambda^2\)

Expect \(\sin 2\beta_{\phi K} \simeq \sin 2\beta_{\psi K}\) in SM at \(O(\lambda^2) \sim 5\%\) level

Bound \(V_{ub}V_{us}^*\) term using only SU(3) & data \(\Rightarrow\) at present

\(\Delta(\sin 2\beta) < 0.3\) \hspace{1cm} (Grossman, ZL, Nir, Quinn, hep-ph/0303171)

\(\psi K_S\): NP could enter through only \(q/p\)

\(\phi K_S\): NP could enter through both \(q/p\) and \(\bar{A}/A\)

Measuring same angle in decays sensitive to different short distance physics may be the key to finding deviations from the SM!
**$B \rightarrow \phi K_S$ — present status**

BABAR and BELLE:

\[
\begin{align*}
S_{\psi K} &= 0.734 \pm 0.054 \\
C_{\psi K} &= 0.026 \pm 0.020 \\
S_{\phi K} &= -0.39 \pm 0.41 \\
C_{\phi K} &= 0.56 \pm 0.43
\end{align*}
\]

$S$ terms differ by: 2.7σ

Need more data to tell...

(Smaller difference in $\eta' K_S$ and $K^+ K^- K_S$ modes)
$B_s \rightarrow \psi\phi$ and $B_s \rightarrow \psi\eta^{(i)}$

- Analog of $B \rightarrow \psi K_S$ in $B_s$ decay — determines the phase between $B_s$ mixing and $b \rightarrow c\bar{c}s$ decay, $\beta_s$, as cleanly as $\sin 2\beta$ from $\psi K_S$

$\beta_s$ is a small $O(\lambda^2)$ angle in one of the “squashed” unitarity triangles

$\psi\phi$ is a VV state, so the asymmetry is diluted by the CP-odd component

$\psi\eta^{(i)}$, however, is pure CP-even

- Large asymmetry ($\sin 2\beta_s > 0.05$) would be clear sign of new physics
\[ B_s \rightarrow D_s^{\pm} K^{\mp} \quad \text{when} \quad |f\rangle \neq |f_{CP}\rangle \]

- Interference of \( B_s \) and \( \bar{B}_s \) decay; clean because single weak phase in each decay

Four amplitudes:
\[
\begin{align*}
\bar{B}_s & \xrightarrow{A_1} D_s^+ K^- \quad (b \rightarrow c\bar{u}s), \\
B_s & \xrightarrow{A_1} D_s^- K^+ \quad (\bar{b} \rightarrow \bar{c}u\bar{s}), \\
\bar{B}_s & \xrightarrow{A_2} K^+ D_s^- \quad (b \rightarrow u\bar{c}s), \\
B_s & \xrightarrow{A_2} K^- D_s^+ \quad (\bar{b} \rightarrow \bar{u}c\bar{s})
\end{align*}
\]

\[
\frac{\bar{A}_{D_s^+ K^-}}{A_{D_s^+ K^-}} = \frac{A_1}{A_2} \left( \frac{V_{cb}V_{us}^*}{V_{ub}V_{cs}^*} \right), \quad \frac{\bar{A}_{D_s^- K^+}}{A_{D_s^- K^+}} = \frac{A_2}{A_1} \left( \frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*} \right)
\]

Magnitudes and relative strong phase of \( A_1 \) and \( A_2 \) drop out if four time dependent rates are measured \( \Rightarrow \) no hadronic uncertainty:

\[
\lambda_{D_s^+ K^-} - \lambda_{D_s^- K^+} = \left( \frac{V_{tb}V_{ts}}{V_{tb}V_{ts}^*} \right)^2 \left( \frac{V_{cb}V_{us}^*}{V_{ub}V_{cs}^*} \right) \left( \frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*} \right) = e^{-2i(\gamma - 2\beta_s - \beta_K)}
\]

- Similarly, \( B_d \rightarrow D^{(*)\pm} \pi^{\mp} \) determines \( \gamma + 2\beta \):

\[
\lambda_{D^+\pi^-} - \lambda_{D^-\pi^+} = e^{-2i(\gamma + 2\beta)}
\]

... ratio of amplitudes is \( \mathcal{O}(\lambda^2) \) \( \Rightarrow \) expected asymmetries are small
Theory progress: Soft-Collinear Effective Theory

- A new EFT to describe the interactions of energetic but low invariant mass particles with soft quanta [“the” connection between heavy quarks and jet physics?]
- Operator formulation instead of studying regions of Feynman diagrams
- Simplified and new proofs ($B \to D\pi$) of factorization theorems
  
  (Bauer, Pirjol, Stewart, ...)

- E.g., $B \to \pi\ell\bar{\nu}$ form factor: Issues: tails of wave fn’s, Sudakov suppression, etc.

Recently proven: $F(Q) = f^{\text{fact.}}(Q) + f^{\text{non-fact.}}(Q)$ — two terms arise in SCET from matrix elements of distinct operators between the same states
Many important omissions

Model independent determination of $|V_{ub}|$ — “hardest” side of UT
(First results from BABAR and BELLE using methods that may lead to small error)

Rare decays — many observables, sensitive to different NP
(Learned that deviations from SM in $b \to s\ell^+\ell^-$ is also $\lesssim 30\%$)

$\gamma$ from $B^{\pm} \to K^{\pm} (D^0, \overline{D}^0) \to K^{\pm} f_i$, or from $B \to D^{(*)\pm} \pi^\mp$

$\alpha$ from $B \to \pi\pi$ isospin analysis, or from $B \to \rho\pi$ Dalitz plot analysis

etc.

Very broad program — independent measurements are searching for NP!
A (near future) best buy list

Many important results expected in the coming years: (apologies for omissions!)

- $|V_{td}/V_{ts}|$: Tevatron is expected to measure $B_s$ mixing (2004–2005?)
- $\beta$: reduce error in $\phi K_S$, $\eta' K_S$, and $KKK$ modes
- $\beta_s$: is CPV in $B_s \rightarrow \psi\phi$ small?
- $|V_{ub}|$: reaching $< 10\%$ will be significant (need to better understand $|V_{cb}|$, too; could be BABAR/BELLE measurements unmatched by LHCb/BTeV)
- Rare decays: $B \rightarrow X_s \gamma$ near theory limited; $q^2$ distribution in $B \rightarrow X_s \ell^+\ell^-$ will be very interesting
- $\gamma$: Need to try all clean modes: $B \rightarrow D(\ast)^\pm \pi^\mp$, $B^\pm \rightarrow DK^\pm$ variants, etc.
- $\alpha$: How small is $\mathcal{B}(\pi^0\pi^0)$; how big are other resonances in $\pi^+\pi^-\pi^0$ Dalitz plot?
- Search for null observables, $a_{CP}(b \rightarrow s\gamma)$, enhanced $B \rightarrow \ell^+\ell^-$, $B \rightarrow \ell\nu$, etc.

Many measurements will not become theory limited by $\sim 2010$!
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At last, the field is now experiment driven!
Extra slides
There are tree and penguin amplitudes, just like for $\psi K_S$

"Tree" ($b \to u\bar{d}d$): \[ \overline{A}_T = V_{ub}V_{ud}^* A_{u\bar{d}} \]

"Penguin": \[ \overline{A}_P = V_{tb}V_{td}^* P_t + V_{cb}V_{cd}^* P_c + V_{ub}V_{ud}^* P_u \]

unitarity: \[ \overline{A}_{\pi^+\pi^-} = V_{ub}V_{ud}^* [A_{u\bar{d}} + P_u - P_t] + V_{cb}V_{cd}^* [P_c - P_t] \]

same as Tree phase not suppressed

Two amplitudes with different weak- and possibly different strong phases; their values not known model independently

Define $P$ and $T$ by: \[ \overline{A}_{\pi^+\pi^-} = T(V_{ub}V_{ud}^*) + P(V_{cb}V_{cd}^*) \]

Ratio of $K\pi$ and $\pi\pi$ rates: \[ |P/T| \sim 0.2 - 0.4 \text{, i.e., } |P/T| \ll 1 \]

Possible solutions: (1) eliminate $P$; or (2) attempt to calculate $P$
$B \rightarrow \pi\pi$ — present status
\[ B^{\pm} \rightarrow (D^0, \bar{D}^0) K^{\pm} \rightarrow f_i K^{\pm} \]

- **B^{\pm} \rightarrow K^{\pm} D**: theoretically clean, experimentally very hard

\[
\frac{|A(B^+ \rightarrow K^+ D^0)|}{|A(B^+ \rightarrow K^+ \bar{D}^0)|} \sim \frac{|V_{ub}|}{\lambda |V_{cb}|} \frac{1}{N_c}
\]

- **B^{\pm} \rightarrow K^{\pm}(D^0, \bar{D}^0) \rightarrow K^{\pm} f_i \quad (i = 1, 2, \text{ at least})**

Use (and determine) final state interaction in D decay in the analysis

Idea: \( B^+ \rightarrow K^+ \bar{D}^0 \rightarrow K^+ f_i \) in doubly Cabibbo-suppressed \( \bar{D}^0 \) decay

\( B^+ \rightarrow K^+ D^0 \rightarrow K^+ f_i \) in Cabibbo-allowed \( D^0 \) decay (e.g., \( f_i = K^- \pi^+ / \rho^+ \))

- It may be better to consider singly Cabibbo-suppressed D decays, \( D \rightarrow K^{\pm} K^{*\mp} \)

Less sensitive to \( D - \bar{D} \) mixing

(Gronau-Wyler)

(Atwood, Dunietz, Soni)

(Grossman, ZL, Soffer, hep-ph/0210433)